# Simulation of the Sensor Network Configuration for the Removal of Ambiguousness in Determination of Coordinates in Passive Location

# Vladimir Orlov

Automation Department, Odessa National Maritime Academy 8, Didrikhson str., Odessa, 65029, Ukraine vorlov@ukr.net

### Abstract

The task under investigation is determination of disposition of emission source according to the temporal delays of the signals accepted in the network of sensors diversity. The criterion of efficiency for optimization of placing of sensors and the standard examples of network configuration are worked out.

# Keywords

Passive Location; Sensor Network; Ambiguity; Determination of Coordinates

# Introduction

Source localization by means of passive location systems is one of the central problems in sonar, navigation, wireless communication, geophysics and many others. Thus, important attention is paid to a choice of criteria and methods of combining the data for increasing the radiating sources location efficiency, taking into account a spatial configuration of a sensors network [1-3].

Nowadays the algorithms of radiation sources coordinates determination based on measurement of temporal time delays between signals, arriving on spatially delivered sensors are widely applied [1-5]. In case of remote sources of radiation it is supposed that the wave front is close to plane and coordinates are calculated by triangulation methods [2, 3]. However in conditions when the extent of basis is commensurable with the distance to a radiation source, the wave front is spherical. It leads to creation of the mathematical model, that is containing system of equations, nonlinearity of which generates several decisions and ambiguity in case of determination of the radiation source position [4-5].

## Simulation of the Sensor Network

The present investigation is devoted to research of influence of a spatial configuration of a sensor network on the accuracy of signal radiation source coordinates calculation in the controlled zone.

It is supposed, that false peak signals are absent, mutually correlative functions of processes on outputs of the selected couples of sensors calculated correctly, with them the temporal signal delays  $T_{i,j}$ ,  $T_{k,m}$  between pairs of sensors are calculated i and j, k and m. Calculation of coordinates (x,y) of a source of radiation which is carried out on the basis of the decision of system of the non-linear (hyperbolic) equations which have been worked out on temporal time delays of found signals from L pairs of sensors is necessary.

The most known model of system of direction finding for determination of coordinates of the source of radiation consists of the L =2 bases containing 2 sensors everyone [2]. The decision (x, y) in Cartesian coordinates about location of the source of radiation in case of arbitrary known coordinates of sensors  $x_i, y_i, i=1,...,4$  is defined from relation of temporal time delays  $T_{1,2}, T_{3,4}$  of each of couples, and distances  $r_1, r_2, r_3, r_4$  from a radiation point to sensors  $r_{34} = r_3 - r_4 = c T_{3,4}, r_{12} = r_1 - r_2 = c T_{1,2}$ , where c - the speed of a sound wave. Coordinates (x, y) of object of radiation are described by a mathematical model in the form of system of two hyperbolic equations [6, 7] which have been worked out according to the Pythagorean theorem

$$r_{12} = r_1 - r_2 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2} - \sqrt{(x_2 - x)^2 + (y_2 - y)^2}$$

$$r_{34} = r_3 - r_4 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2} = \sqrt{(x_4 - x)^2 + (y_4 - y)^2}.$$
 (1)

For the method of computation of coordinates on timedifferences-of-arrival (TDOA) some types of the ambiguities exist arising from the following reasons.

- Difference of the discrete space in polar and Cartesian coordinate systems. Thus, one couple temporal time delays  $t = T_{1,2}$ ,  $\tau = T_{3,4}$  in (1) corresponds to several nearby to the located elements of permission (cells)  $t, \tau \to x_i, y_i, i = 1,..., N_{t,\tau}$  in Cartesian coordinate system.
- Direct decision of system (1) method of squaring. Thus there are some decisions generating speckled false points, significantly remote from the coordinate of a source of radiation. In particular, essential ambiguity takes place in case of the analytical decision (1) as a result of double exponentation of the equations in a square. It generates to 4 decisions provided by vectors  $S_{i} = (x_{i}, y_{i})^{T}, i = 1,...,4$ , from which 3 false and don't correspond to the true coordinates of a source of radiation. It is necessary to mark that receiving exact analytical expressions even for the simplest implementation of a network from L=2 bases containing randomly located M = 3 sensors (two couples sensors, one of them the general), results in ambiguity in the form of two decisions [8].

For simplification of calculations of coordinates with a margin error less than a half of the size of an element of permission approach which is based on grid sampling of a card of compliance of temporal time delays  $T_{1,2}, T_{3,4} \rightarrow x, y$  to each point in the Cartesian coordinate system calculated in advance is offered (1). It allows to exclude the need of the decision of system of the hyperbolic equations if the size of a grid of coordinates of the purposes for all possible time delays  $T_{1,2}, T_{3,4}$  taking into account the size of a cell of a controlled zone is limited. It is easy to define the requirements to the size of a grid and to the memory necessary for its storage. An example of a big zone of characteristic monitoring, reconnaissance [2, 9] which is carried out on a square  $D_x = D_y = 10$  km is reviewed. In case of sampling rate of f = 1000 Hz and the speed of a sound wave of v = 330m/s, the size of an element of permission on range

make d = v/f = 0.33 m. Then storage of cells  $N_x N_y = D_x D_y/d^2 \approx 10^9$  of a card of compliance (the cell occupies 8\*4=36 bytes) requires about 36 Gbytes of memory that is implemented on the modern element basis.

For research of ambiguity an example of nonoptimal placement of the network of sensors in a controlled zone (Figure 1) in the form of a square with the side is reviewed  $x_{\rm max}=y_{\rm max}$  =2 m. Bases are located parallely, and steam 1 and 2 of sensors make the first basis of the size  $B_1=B(1,2)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ , and steam 3 and 4 of sensors - the second basis  $B_2=B(3,4)=\sqrt{(x_3-x_4)^2+(y_3-y_4)^2}$ . The audible signal is digitized with sampling rate of f=1000 Hz that at a speed of distribution of a sound wave of 330 m/s corresponds to a permission element on range of d=v/f=0.33 m.

From Figure 1a it is easy to note that any line item of a source of radiation on the shaped line passing through centers of bases, leads to identical time delays of signals  $T_{1,2}=t_1-t_2=0$ ,  $T_{3,4}=t_3-t_4=0$  (the number of elements is determined by a scale of intensity). Then the set of decisions (1) represents set of all elements located on the shaped line. It generates essential ambiguity in position fix of a source of radiation as the source is only in one cell on this straight line, and remaining decisions are false.

In Figure 1b the card of a zone of monitoring with location of ambiguous elements in Cartesian coordinate system on which sampling of elements of permission is carried out is provided. Multiplicity of ambiguity is determined by a scale of intensity (number of the false decisions corresponding to one couple time delays). Apparently, in case of a time delay  $t = T_{1,2} = 0$  on basis  $B_1$  and  $\tau = T_{3,4} = 0$  on basis  $B_2$  the greatest multiplicity of ambiguity, to equal  $N_{t,\tau} = N_{0,0}$ =13 elements of permission or the size of a controlled zone takes place. In case of other time delays ambiguity is less, and on white squares of coordinate of a source are defined by the single decision. From the analysis of Figure 1a and Figure 1b follows that it is necessary to define the configuration of sensors providing the minimum number of elements of permission, time delays corresponding to everyone couple.

For comparing of configurations of systems it is

necessary to define the criterion of ambiguity considering precision characteristics of an assessment of location of the purpose.

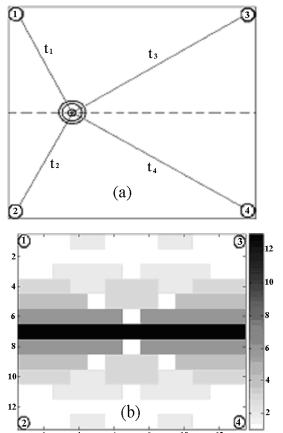


FIG. 1 EXAMPLE OF AMBIGUOUS DETERMINATION OF COORDINATES: CONFIGURATION OF SENSORS AND A RADIATION SOURCE ON THE PLANE OF A ZONE OF MONITORING (A); LOCATION OF AMBIGUOUSLY DEFINED ELEMENTS ON THE PLANE OF A ZONE OF MONITORING (B)

If the decision on signal existence on temporal time delays  $t=T_{_{1,2}}, \tau=T_{_{3,4}}$  is made, in Cartesian coordinate system it can correspond  $N_{_{t,\tau}}>0$  to several points  $S_{_i}(x_{_i},y_{_i}), i=1,...,N_{_{t,\tau}}$ . Mean value and dispersion of an assessment of coordinates are defined by expressions:

$$\overline{S}_{t,\tau} = \sum_{i=1}^{N_{t,\tau}} S_i(x_i, y_i);$$

$$\sigma_{t,\tau}^2 = (N_{t,\tau})^{-1} \sum_{i=1}^{N_{t,\tau}} [\overline{S}_{t,\tau} - S_i(x_i, y_i)]^2. \tag{2}$$

In case of the single decision parameters of delay correspond to  $t, \tau \to x, y$  one element of permission (cell) the minimum error  $\sigma_{t,\tau}$  =0 is reached. With ambiguity when to one couple temporal time delays

 $t, \tau \to x_i, y_i, i = 1,..., N_{t,\tau}$  corresponds two and more close cells in Cartesian coordinate system, takes place

 $\sigma_{t,\tau}$  >0, and value of ambiguity is equivalent to lowering of resolution capability and accuracy of an assessment of location of the purpose. If cells don't adjoin, mean squared error (MSE)  $\sigma_{t,\tau}$  significantly increases that corresponds to appearance of false targets.

These features allow to evaluate numerically ambiguity and to define efficiency of placement of sensors, as the amount of the errors enveloping possible provisions of the purpose. As criterion of ambiguity the error square, equal to the amount of dispersions on all possible time delays in a controlled zone is accepted:

$$\sigma^{2} = \sum_{t=1}^{T_{1 \max}} \sum_{\tau=1}^{T_{2 \max}} W_{t,\tau} \sigma_{t,\tau}^{2} , \qquad (3)$$

where  $W_{t,\tau}$  - a weight multiplier of importance of the cells enveloped by time delays  $t,\tau$ ;  $T_{i,\max} = B_i/c, i = 1,2$  - the maximum time delays of signals on bases  $B_i$  i - pairs of sensors.

Then determination of a configuration M of sensors is brought together to the choice of coordinates of sensors in the form of vectors X,Y of the size M and to the choice of bases  $B_i$ , i=1,L on pairs of sensors. The task of optimization is brought together to functionality minimization from MSE of an error and looks like

$$\min_{X,Y} \sigma(X, Y, B_1, B_2, ..., B_L)$$
 (4)

in case of restrictions  $B_i \leq B_{\max}$ , i = 1, L;  $x_i \leq x_{\max}$ ,  $y_i \leq y_{\max}$ , i = 1, M on the maximum sizes: bases  $B_i$  and controlled zone  $x_{\max}$ ,  $y_{\max}$ .

The optimum is reached as a result of integer search of all values X,Y and bases  $B_i$ , i=1,L on a grid of a controlled zone with a permission element d=v/f. Generally, the maximum possible quantity of bases is defined by number of sensors L=M(M-1)/2, and the mathematical model of system of direction finding (1) becomes complicated and contains to the L equations

$$r_{ij} = r_i - r_j = cT_{ij} = \sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_j - x)^2 + (y_j - y)^2}$$

$$i, j = 1, 2, ..., M; \quad i \neq j. \quad (5)$$

In practice the situation in which the configuration is set often takes place, and it is necessary to select pairs of sensors for bases in case of which ambiguity minimum.

Let's begin efficiency research on the configuration containing the minimum quantity of sensors M =3, necessary for purpose position fix. In Figure 2a the configuration "angle" on the sides of a square zone in which two bases are applied is shown, first of which contains 1 and the 2nd sensors  $B_1 = B(1,2)$ , the second -2 and the 3rd sensors  $B_2 = B(2,3)$ .

Apparently, zones of ambiguity contain to 3 elements of permission and concentrate in 3 zones. Increase of efficiency is possible at the expense of use of all possible bases on this configuration of sensors. The result of application of the third additional basis for which it is necessary to enter the third equation in (1) is given in Figure 2b. It significantly reduces ambiguity and the MSE decreases by 4 times. In Figure 2c ambiguity zones for system with the high resolution, different system in Figure 4 sampling rate increase by 5 times are provided. From their comparing follows that the same circuits of ambiguity that allows to conduct researches on models with low resolution and, thereby, to reduce computing costs of search of an optimum configuration remain.

In Figure 3 examples of placement of 4 sensors on a configuration "cross" are given  $B_{1} = B(1,3)$ ,  $B_2 = B(2,4)$ . For the system provided in Figure 3a, bases are square diagonals. In this case the extent of basis exceeds the side of a square of the zone of monitoring that allows to exclude completely ambiguity and  $\sigma$ =0. If sensors are distributed on the circle inscribed in a square (Figure 3b), the extent of basis decreases to the side of a square and ambiguity at the edges of a monitoring zone takes place. The essential increase in ambiguity arises in case of small bases of rather controlled zone (Figure 3c). In practice such situation corresponds to use of systems of detection of snipers for which microphones are placed on a helmet of the security guard with bases about 0,2 meters [10], and the size of a zone of monitoring occupies hundreds meters. From Figure 3c follows that the worst case situation corresponds to location of the shooter on square diagonals, only the direction of a

source of a shot thus is defined to within  $9^0$ .

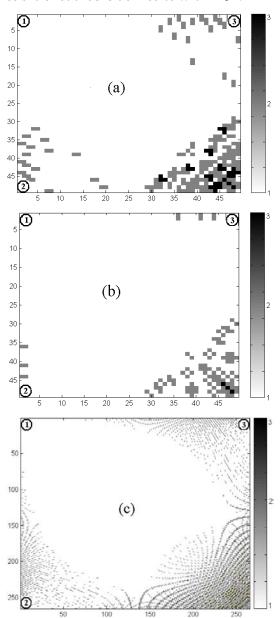
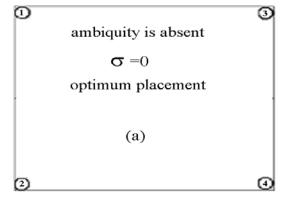


FIG. 2 EXAMPLES OF PLACEMENTS OF SENSORS AND AMBIGUITY ZONES FOR CONFIGURATIONS OF THE BASES MADE ON 3 SENSORS: APPLICATION OF TWO BASES (A); THREE BASES (B); THREE BASES AND THE INCREASED SAMPLING RATE OF A SIGNAL (C)



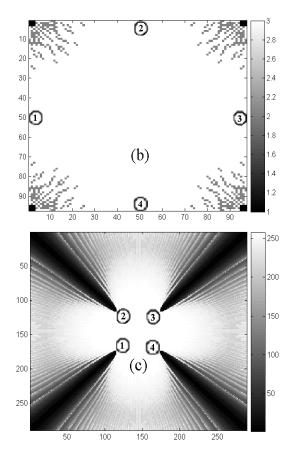
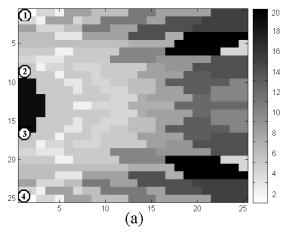


FIG. 3. EXAMPLES OF PLACEMENTS OF SENSORS AND AMBIGUITY ZONES FOR CONFIGURATIONS OF THE BASES MADE ON 4 SENSORS: A CONFIGURATION "CROSS" APPLICATION OF TWO BASES (A); SENSORS ARE DISTRIBUTED ON A CIRCLE (B); CONFIGURATION WITH THE REDUCED BASES (C)

Example of other placement of 4 sensors is the configuration "line" with the even stride which optimization is illustrated in Figure 4. Apparently, the case of use of L =2 two bases  $B_1 = B(1,2)$  and  $B_2 = B(3,4)$ , provided in Figure 4a, possesses several shortcomings connected to the small extent with the bases and their nonoptimal placement in the zone of monitoring.



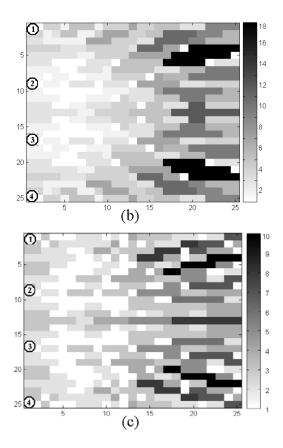


FIG. 4 EXAMPLES OF CONFIGURATIONS OF THE BASES LOCATED ON THE LINE: TWO BASES (A); THREE BASES (B); FOUR BASES (C)

Optimization at the expense of a choice of bases without change of position of sensors is performed.

Application of the third basis (Figure 4b) L=2 look allows  $B_3 = B(2,3)$  to reduce an error by 4 times, the fourth basis (Figure 4c) L=4  $B_4 = B(1,4)$  – by 1,5 times. It is necessary to mark that for big zones of monitoring, for example, in tasks of artillery acoustic reconnaissance [2] with zones to 10-30 km, application of big bases of a look isn't always possible  $B_4 = B(1,4)$ . It is connected with possible lowering of correlation of signals because of distortions of the signals entered by different propagation paths to sensors, and also to relation reduction signal/noise on inputs of sensors.

# Conclusions

The conducted researches on calculations of location of radiation source allow to draw the following outputs:

 The criterion is developed and the task of optimization of the sensor network configuration taking into account restrictions on the sizes of a controlled zone is formulated;  It is set that ambiguity minimum in case of execution of the following requirements to arrangement of sensors: application of the greatest possible number of bases and their sizes, support of orthogonality of lines of bases by means of approximation to a configuration "cross".

The considered method of mathematical simulation of a configuration allows to realize reasonably creation of a sensor network and to predict precision characteristics of a passive location in a controlled zone.

#### REFERENCES

- Ákos Lédeczi, András Nádas, Péter Völgyesi, György Balogh, Branislav Kusy, János Sallai, Gábor Pap, Sebestyén Dóra, Károly Molnár, Miklós Maróti, Gyula Simon. "Countersniper system for urban warfare." ACM Transactions on Sensor Networks 1 (2005): 153 – 177.
- Carrapezza E.M. et al. "DARPA Counter Sniper Program Phase I Acoustic Systems Demonstration Results." SPIE 2938 (1997): 299-310.
- Duckworth, Gregory L.; Barger, James E.; Carlson, S. H.; Gilbert, Douglas C.; Knack, M. L.; Korn, J.; Mullen, R. J. "Acoustic Counter-Sniper Systems for Law Enforcement." Proc. SPIE Sensors, C3I, Information, and Training Technologies for Law Enforcement 3577 (1999): 210-230.
  - http://www.raytheonbbn.com/resources/pdf/sniper\_spie 98.pdf.
- http://www.mdpi.com/1424-8220/12/10/13781/pdf
- Jose Velasco, Daniel Pizarro and Javier Macias-Guarasa.

  "Source Localization with Acoustic Sensor Arrays Using
  Generative Model Based Fitting with Sparse
  Constraints." Sensors 12 (2012): 13781-13812.
- Kuriksha A. A. "Analysis of uniqueness and accuracy of an assessment of angular data of a source in the low-element interferometer." Interstate joint-stock

- corporation "Vympel". Scientific articles. http://www.vimpel.ru/analiz1.htm.
- Malcolm Hawkes, Arye Nehorai. "Effects of Sensor Placement on Acoustic Vector-Sensor Array Performance." IEEE Journal of Oceanic Engineering 24 (1999): 33-40.
- Orlov V. V. "Detection of location of a source of radiation on the basis of correlative spatial processing." Radioyelektronika. Informatics. Control 36 (2003): 42-46.
  - $Http://e\,wdte\,st.com/ri/ri\_2007\_1/ri\_2007\_1.pdf$
- Rappaport T. S., Reed J. H., Woerner B. D. "Position Location using Wireless Communications on Highways of the Future." IEEE Communications Magazine 34 (1996): 33-41.
- Spiesberger, J.L., "Hyperbolic location errors due to insufficient number of receivers." Journal of the Acoustical Society of America 109 (2001): 3076-3079.
- Y.T. Chan, K.C. Ho. "A Simple and Efficient Estimator for Hyperbolic Location." IEEE Transactions on Signal Processing 42 (1994): 1905-1915.



Vladimir V. Orlov was born in 1957 in Ukraine. He got the university degree in the field of radio engineering in Odessa National Polytechnical University in 1980. He received the Ph.D. degree in the field of radar and a radio navigation in 1994. In 1980 - 2012 he worked in the scientific

organizations on creation of location systems of different applications: radar, sonar and network of acoustic sensors. He has more than 80 publications. The activity list is provided on the sites: www.orlov-v.narod.ru, www.orlov.xe0.ru. His current research interests include adaptive systems of detection, recognition and cleaning of signals in the conditions of clutters.